

2. **PROBLEM 2.** (10 points) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation
- $$f(x+y) = f(x) + f(y) \quad \text{for all } x, y \in \mathbb{R}.$$
- Suppose that f is continuous at $x=0$. Prove that f is linear, i.e., $f(x) = cx$ for some constant $c \in \mathbb{R}$.
3. **PROBLEM 3.** (10 points) Let n be a positive integer. Consider the sequence of numbers
- $$a_1, a_2, \dots, a_n$$
- defined by
- $$a_k = \frac{1}{k} \quad \text{for } k = 1, 2, \dots, n.$$
- Prove that the sum of the reciprocals of the first n positive integers is less than 2.
4. **PROBLEM 4.** (10 points) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation
- $$f(x+y) = f(x)f(y) \quad \text{for all } x, y \in \mathbb{R}.$$
- Suppose that f is continuous at $x=0$ and $f(0) = 1$. Prove that $f(x) = e^{cx}$ for some constant $c \in \mathbb{R}$.
5. **PROBLEM 5.** (10 points) Let n be a positive integer. Consider the sequence of numbers
- $$a_1, a_2, \dots, a_n$$
- defined by
- $$a_k = \frac{1}{k^2} \quad \text{for } k = 1, 2, \dots, n.$$
- Prove that the sum of the reciprocals of the squares of the first n positive integers is less than 2.
6. **PROBLEM 6.** (10 points) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation
- $$f(x+y) = f(x) + f(y) + c \quad \text{for all } x, y \in \mathbb{R},$$
- where c is a constant. Suppose that f is continuous at $x=0$. Prove that $f(x) = cx + d$ for some constants $c, d \in \mathbb{R}$.
7. **PROBLEM 7.** (10 points) Let n be a positive integer. Consider the sequence of numbers
- $$a_1, a_2, \dots, a_n$$
- defined by
- $$a_k = \frac{1}{k^3} \quad \text{for } k = 1, 2, \dots, n.$$
- Prove that the sum of the reciprocals of the cubes of the first n positive integers is less than 2.
8. **PROBLEM 8.** (10 points) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation
- $$f(x+y) = f(x)f(y) + c \quad \text{for all } x, y \in \mathbb{R},$$
- where c is a constant. Suppose that f is continuous at $x=0$ and $f(0) = 1$. Prove that $f(x) = e^{cx} + d$ for some constants $c, d \in \mathbb{R}$.
9. **PROBLEM 9.** (10 points) Let n be a positive integer. Consider the sequence of numbers
- $$a_1, a_2, \dots, a_n$$
- defined by
- $$a_k = \frac{1}{k^4} \quad \text{for } k = 1, 2, \dots, n.$$
- Prove that the sum of the reciprocals of the fourth powers of the first n positive integers is less than 2.
10. **PROBLEM 10.** (10 points) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation
- $$f(x+y) = f(x) + f(y) + c \quad \text{for all } x, y \in \mathbb{R},$$
- where c is a constant. Suppose that f is continuous at $x=0$ and $f(0) = 1$. Prove that $f(x) = cx + d$ for some constants $c, d \in \mathbb{R}$.

QUESTION: 12. Which of the following is not a type of nonverbal communication?

- Body language
- Facial expressions
- Verbal communication
- Touch

ANSWER: 12. C

- Body language is a type of nonverbal communication. It includes gestures, posture, and facial expressions.
- Facial expressions are a type of nonverbal communication. They can convey a wide range of emotions, from happiness to sadness.
- Verbal communication is a type of communication that involves the use of words. It can be spoken or written.
- Touch is a type of nonverbal communication. It can be used to convey affection, support, or aggression.

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12. a) Eine Funktion $f: \mathbb{R} \rightarrow \mathbb{R}$ ist eine Abbildung, die jedem $x \in \mathbb{R}$ genau ein $y \in \mathbb{R}$ zuordnet. Die Abbildung f ist dann eine Funktion, wenn sie die Eigenschaft hat, dass für jedes $x \in \mathbb{R}$ genau ein $y \in \mathbb{R}$ existiert, so dass $f(x) = y$ gilt.

13. Monotonieverhalten

13. a) Gegeben sei die Funktion $f: \mathbb{R} \rightarrow \mathbb{R}$, die durch $f(x) = x^2 - 4x + 5$ definiert ist. Untersuchen Sie das Monotonieverhalten der Funktion f auf dem Intervall $[0, 6]$.

13. b) Gegeben sei die Funktion $f: \mathbb{R} \rightarrow \mathbb{R}$, die durch $f(x) = x^3 - 3x^2 + 2x$ definiert ist. Untersuchen Sie das Monotonieverhalten der Funktion f auf dem Intervall $[-1, 3]$.

14. Extremwerte

14. Gegeben sei die Funktion $f: \mathbb{R} \rightarrow \mathbb{R}$, die durch $f(x) = x^3 - 3x^2 + 2x$ definiert ist. Bestimmen Sie die lokalen Extremwerte der Funktion f auf dem Intervall $[-1, 3]$.